Image restoration

莊子肇 副教授
中山電機系

Enhancement vs. Restoration

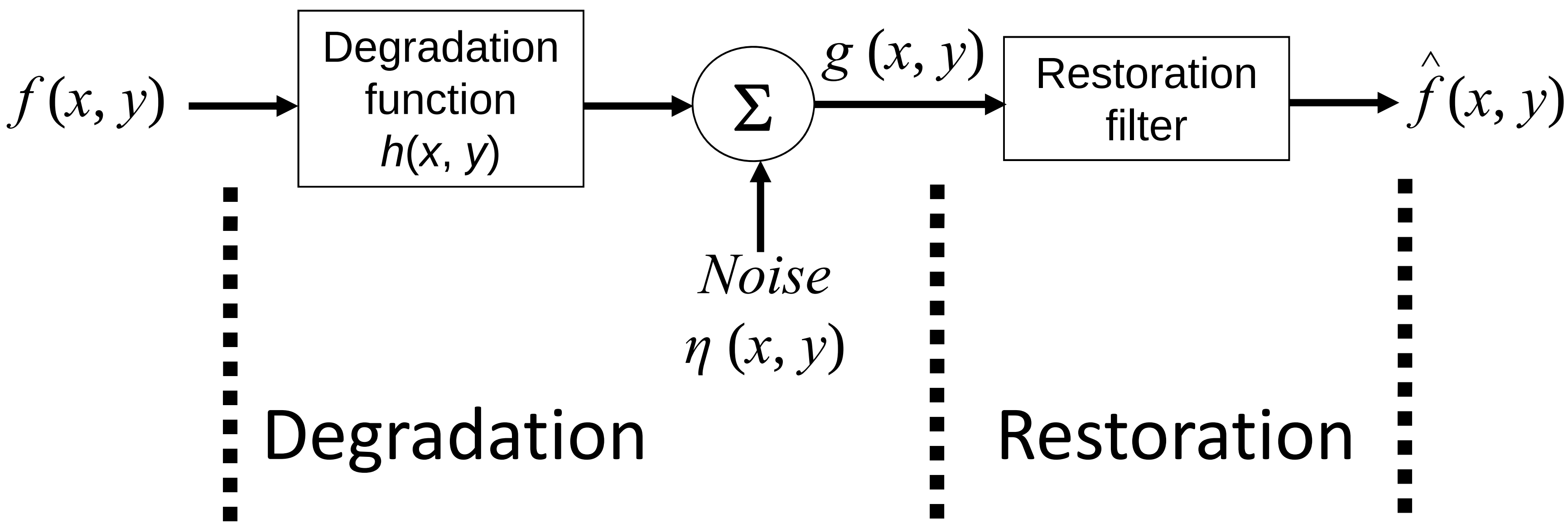
Enhancement		Restoration
"Better" visual representation	⟷	Remove effects of sensing environment
Subjective	⟷	Objective
No quantitative measures	⟷	Mathematical, model dependent quantitative measures

Motivation

- Remove, or at least reduce, blur and noise in a digital image.

- Restoration attempts to recover an image that have been degraded by using a priori knowledge of the degradation function.

Image Degradation/Restoration Model

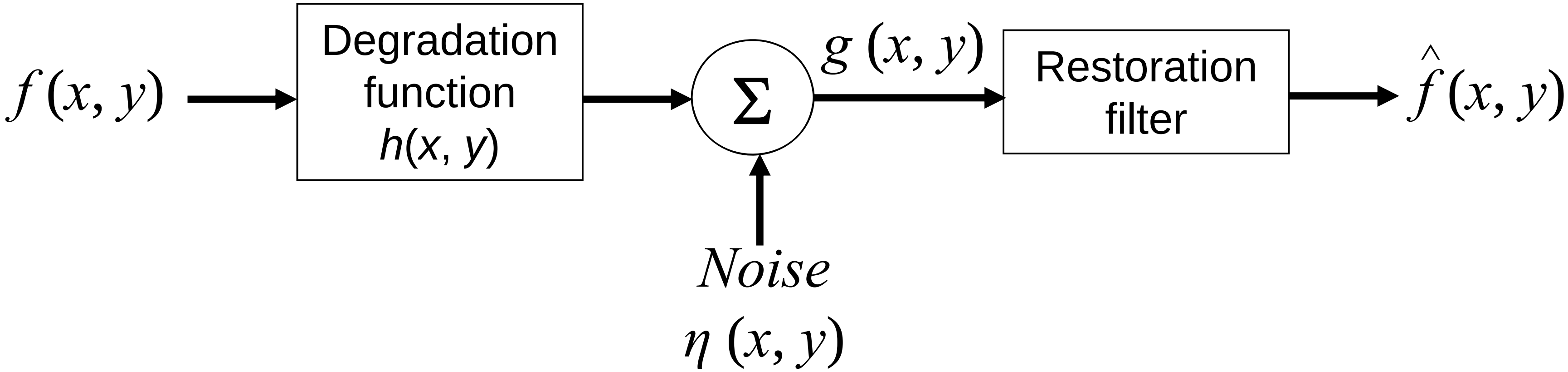


Degradation model with additive noise

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v) \times F(u, v) + N(u, v)$$

Goal of restoration



Goal of restoration: to make $\hat{f}(x, y) \sim f(x, y)$

Noise Models

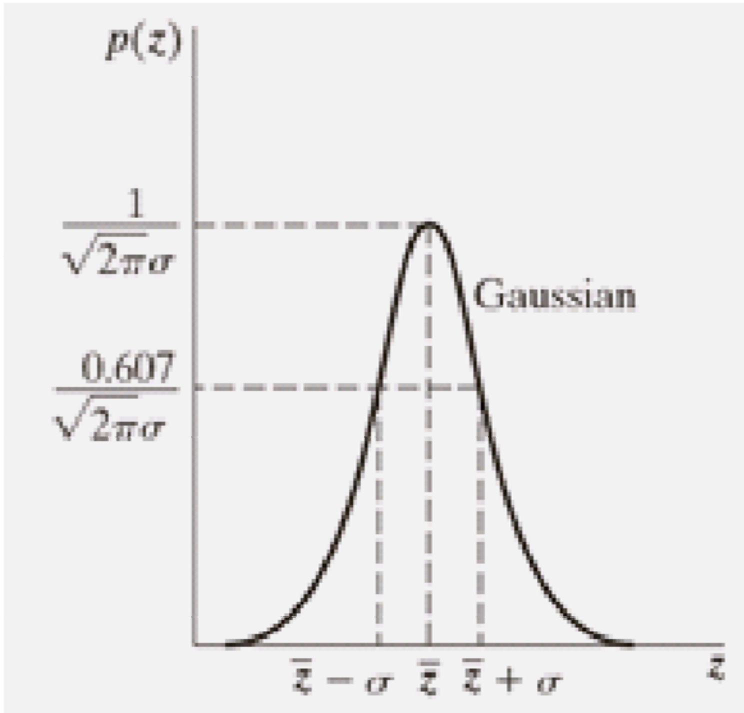
- Noise arises during image acquisition and/or transmission.
- Random Noise
  - Noise is independent of location and time.
  - Can be described by probability density function (PDF)
- White noise
  - Constant power spectral density
  - Zero mean

Gaussian noise (normal noise)

$$p(z)=\frac{1}{\sqrt{2\pi}\cdot\sigma}e^{-(z-\bar{z})^2/2\sigma^2}$$

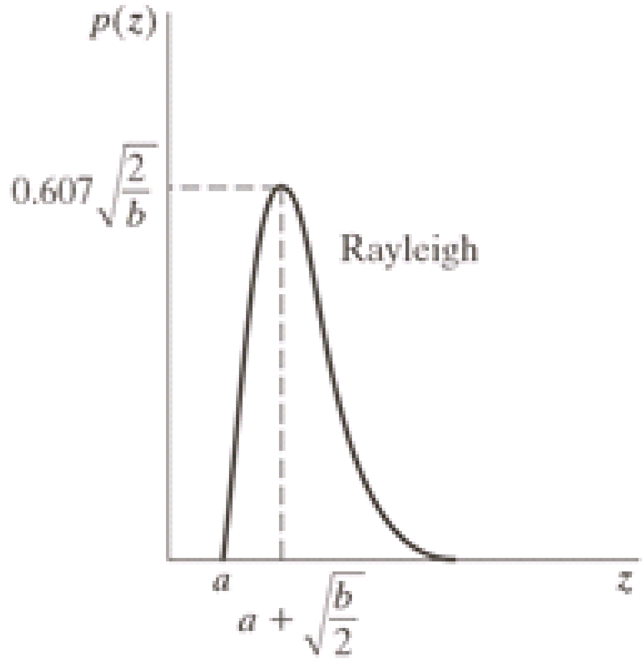
z : intensity of noise

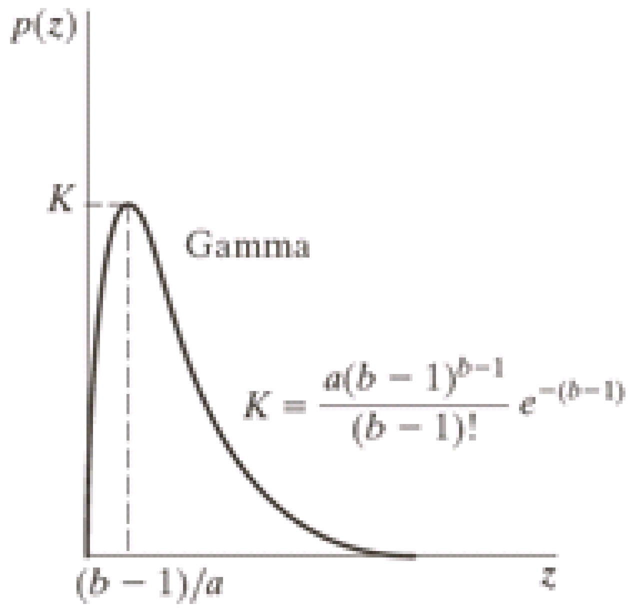
σ : standard deviation



Gaussian noise = white noise ??

More PDFs of noise





Rayleigh noise

When $z \geqq a$, $p(z) = \frac{2}{b}(z-a)e^{-(z-a)^2/b}$

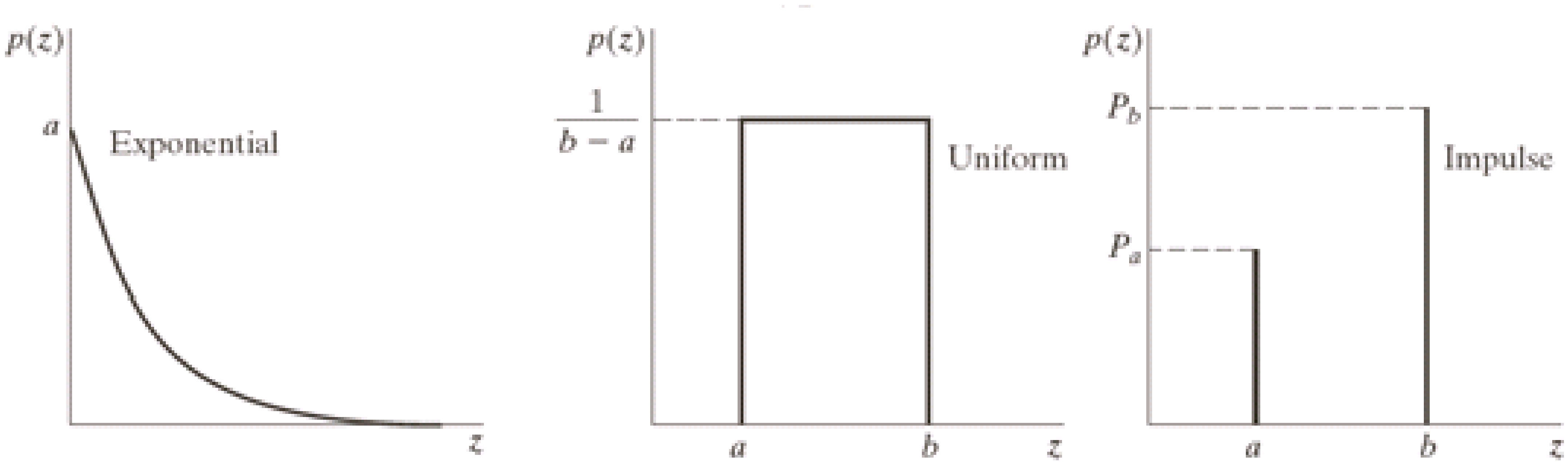
When $z < a$, $p(z) = 0$

Gamma noise

When $z \geqq 0$, $p(z) = \frac{a^b z^{b-1}}{(b-1)!} e^{-az}$

When $z < 0$, $p(z) = 0$

More PDFs of noise



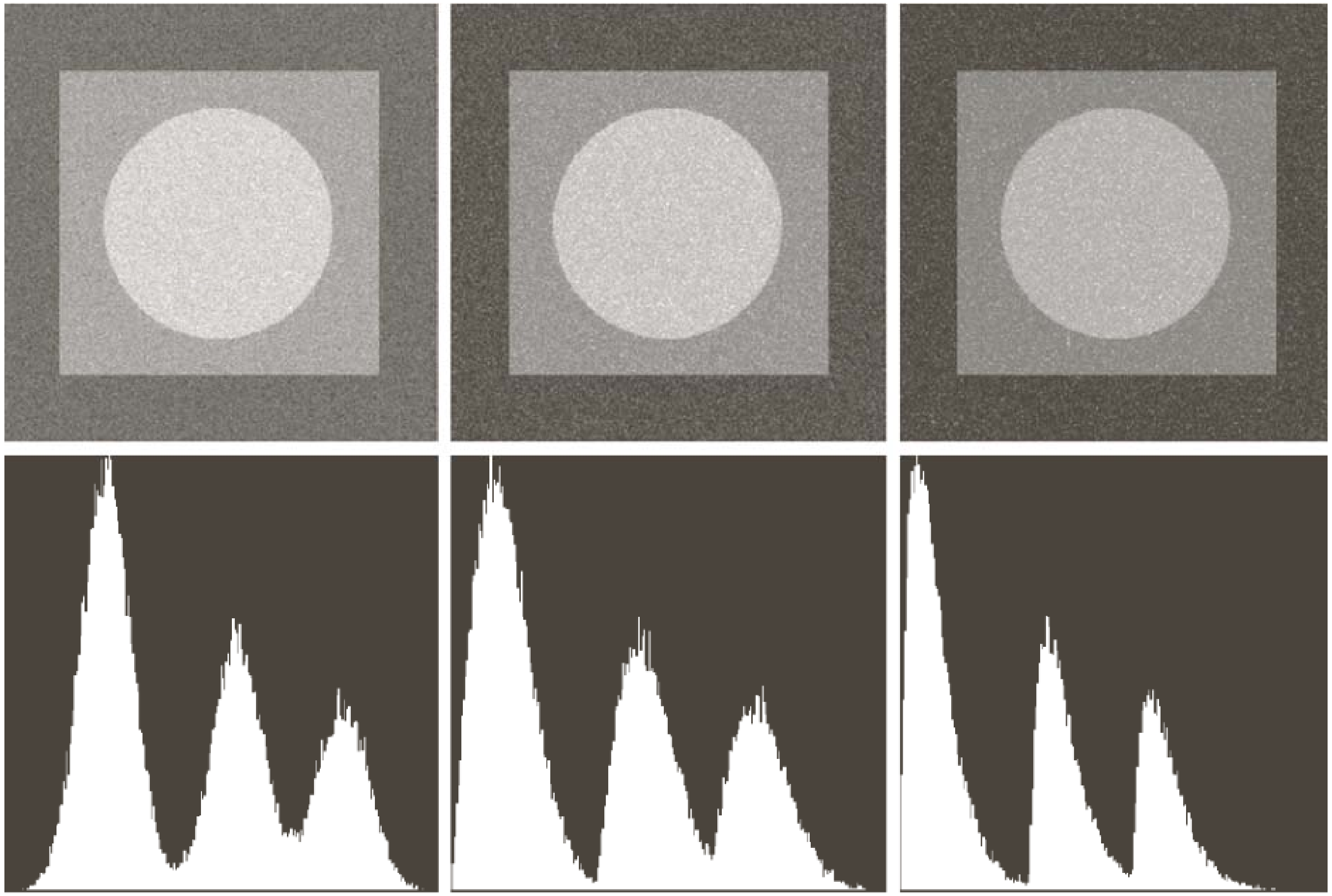
Exponential noise

Uniform noise

Impulse (salt-and-pepper) noise

Patterns of noisy images

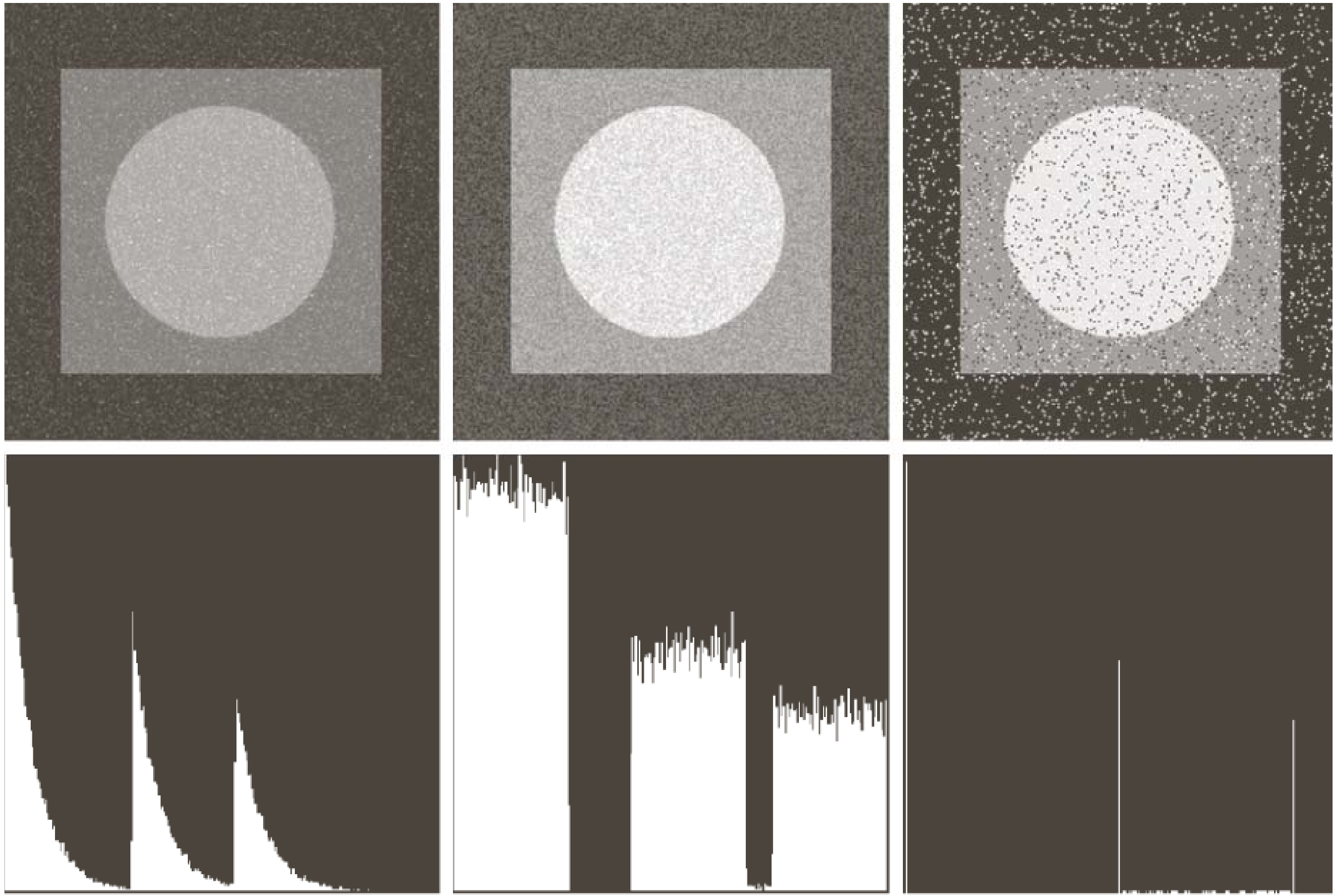
Test pattern



Gaussian Rayleigh Gamma

Patterns of noisy images

Test pattern

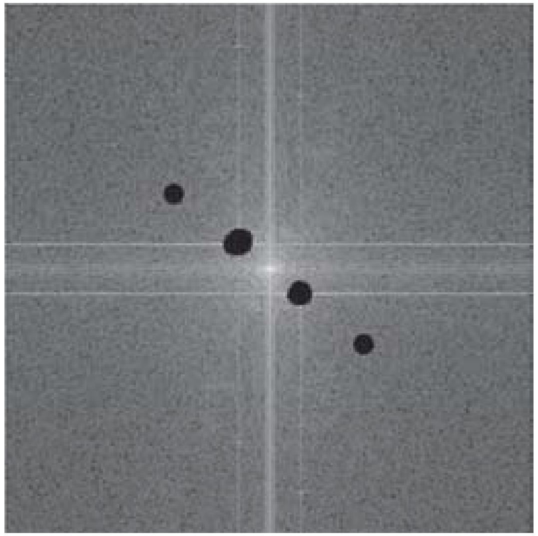


Exponential Uniform Salt & Pepper

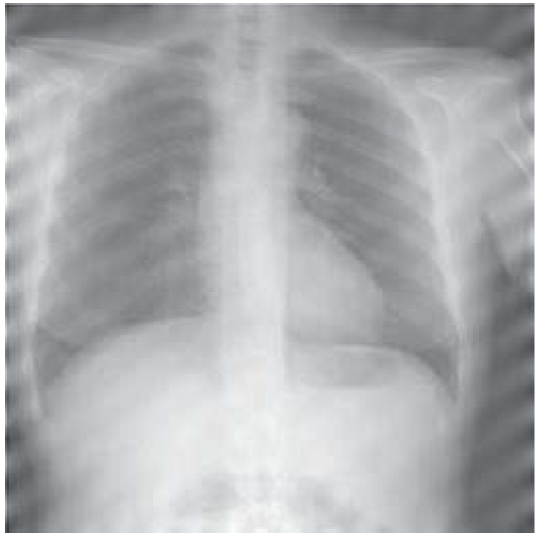
Periodic Noise

- Spatially dependent noise
- Reduced by frequency domain filtering

Degraded X-ray image



Removal by notch filter

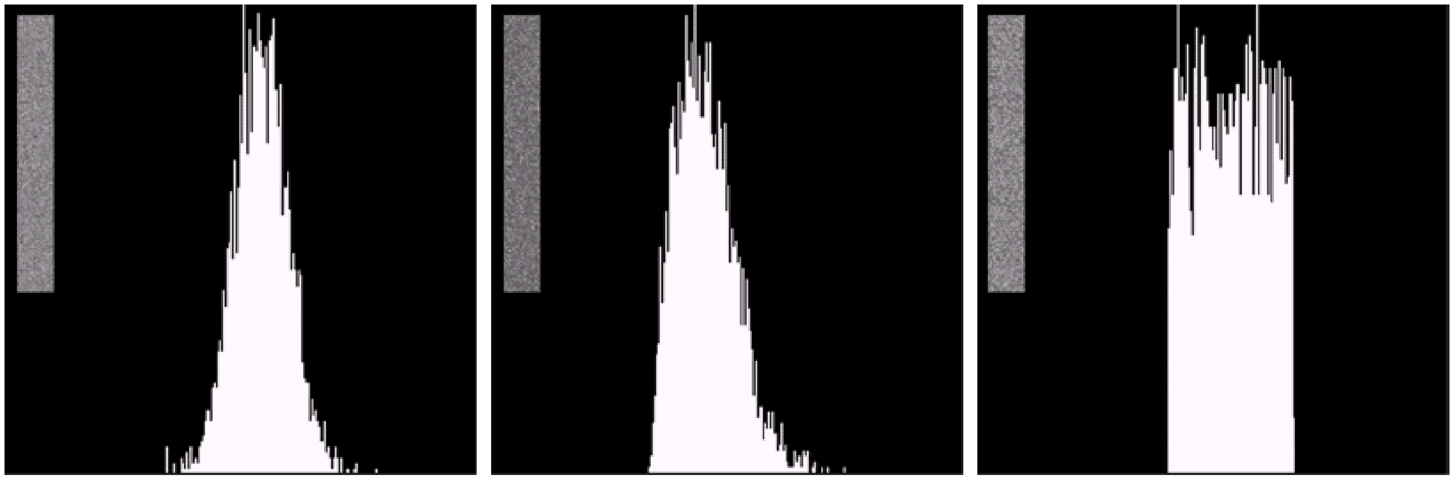


Inverse 2D-FT

Estimate of the Noise Parameters

- 由感測器的特性或規格說明加以評估
- 選定<u>適當區域</u>，運用平均值與標準差

$$\mu = \sum z_i p(z_i) \qquad \sigma^2 = \sum (z_i - \mu)^2 p(z_i)$$



Restoration filters

- Random noise only: spatial filtering
  - Mean filters
  - Order-statistic filters
  - Adaptive filters
- Periodic noise only: spectral filtering
  - Band-reject filters
  - Notch filters
- Non-identity degradation function

Restoration – in Spatial domain

- Assume the only degradation present in an image is noise.

$$g(x,y) = f(x,y) + n(x,y)$$

$$G(u,v) = F(u,v) + N(u,v)$$

Mean filters

- Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

- Geometric mean filter

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Mean filters

a	b
c	d

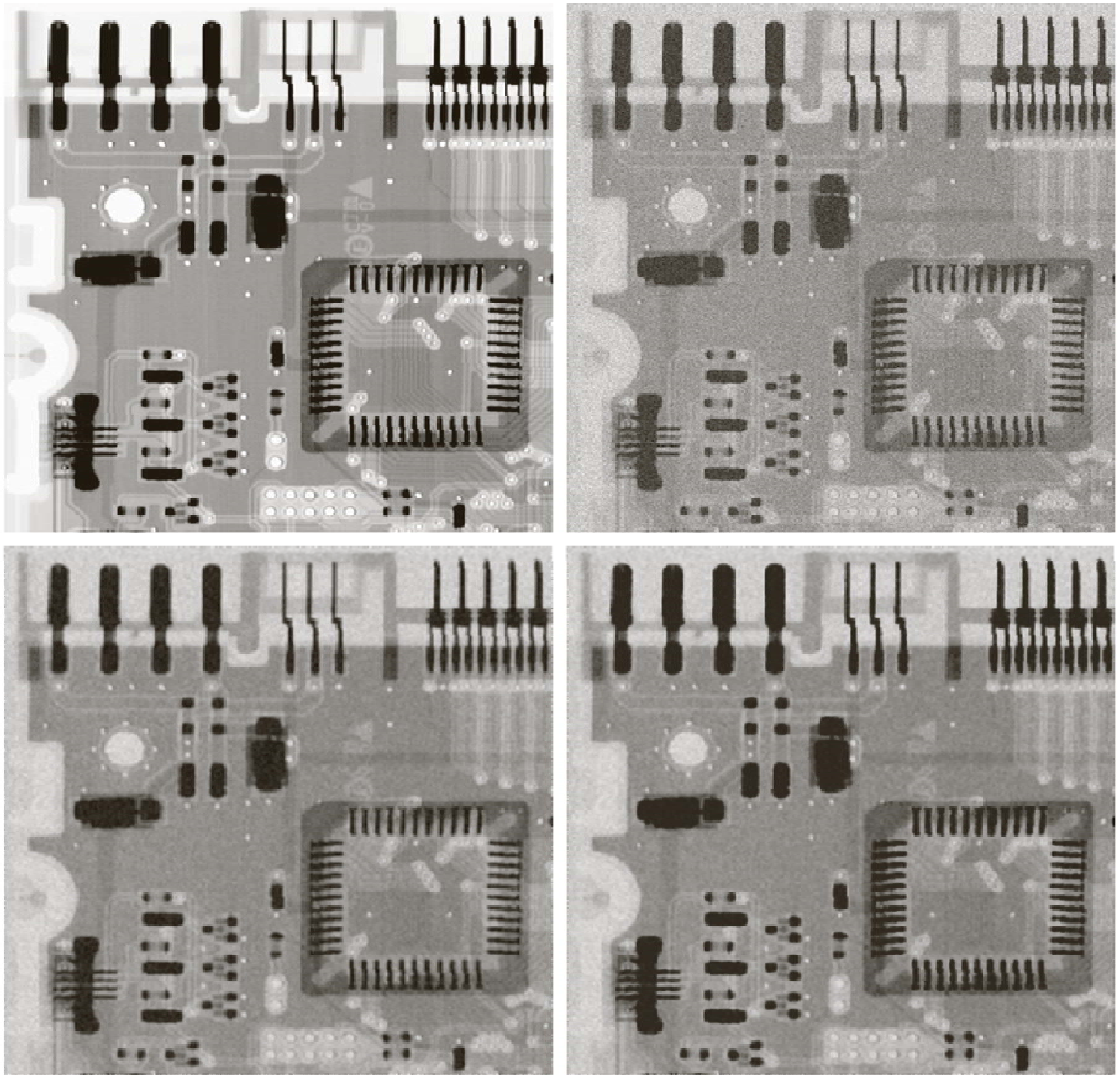
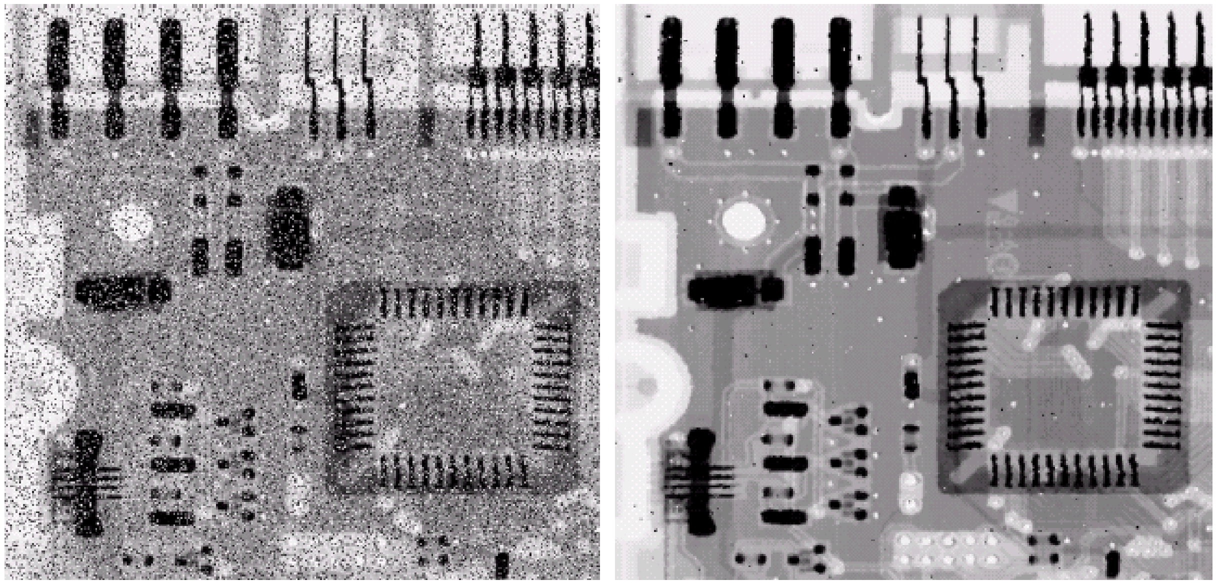


FIGURE 5.7
(a) X-ray image.
(b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size.
(Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Order-Statistics filters

- Median Filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$



除此之外還有: max/min filter, midpoint filter,…

Adaptive filters

- Local noise reduction filter

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

$g(x, y)$: the value of the noisy image at (x, y)
$\sigma_\eta{}^2$: variance of the additive noise
$\sigma_L{}^2$: local variance of pixels in S_{xy}
m_L : local mean of pixels in S_{xy}
S_{xy} : the rectangular filter region for (x, y)

If $\sigma_\eta{}^2 = 0$?? If $\sigma_L{}^2 = \sigma_\eta{}^2$?? If $\sigma_L{}^2 > \sigma_\eta{}^2$??

Adaptive filters

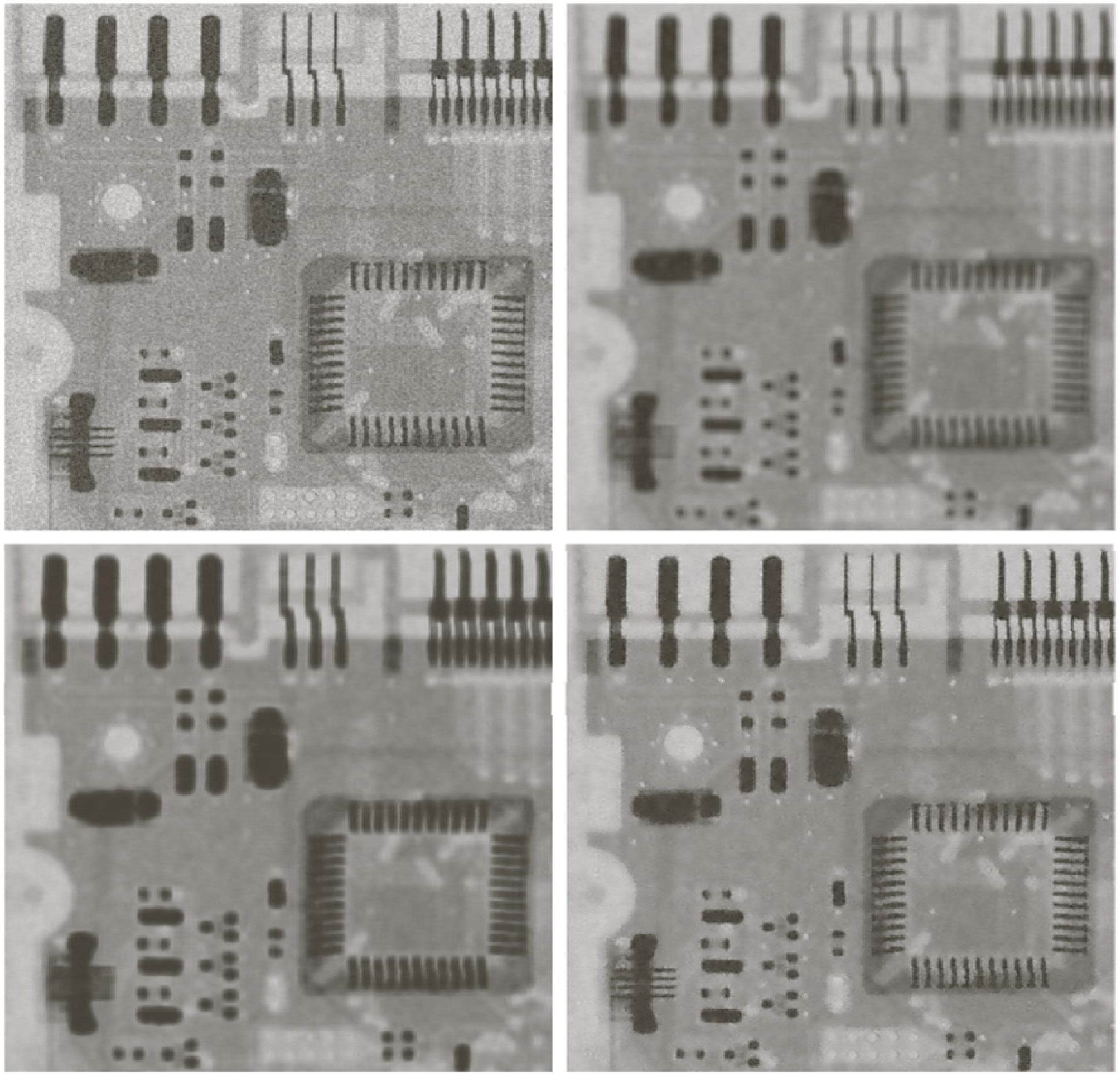
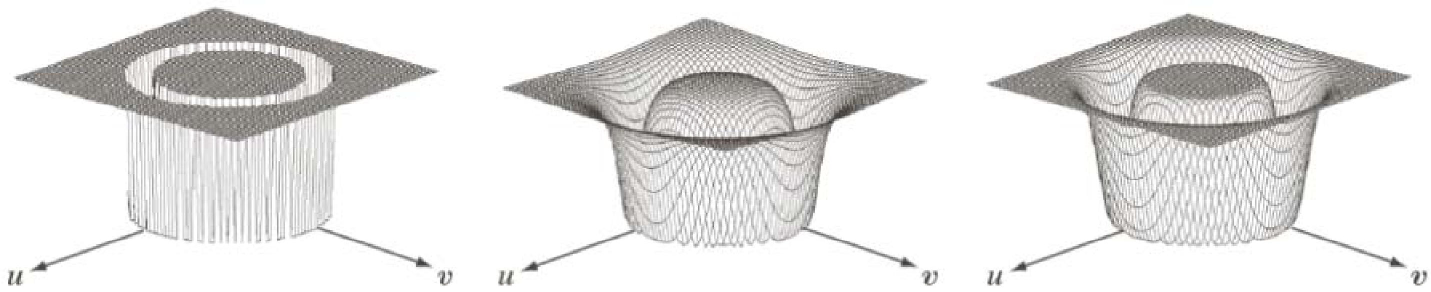


FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.

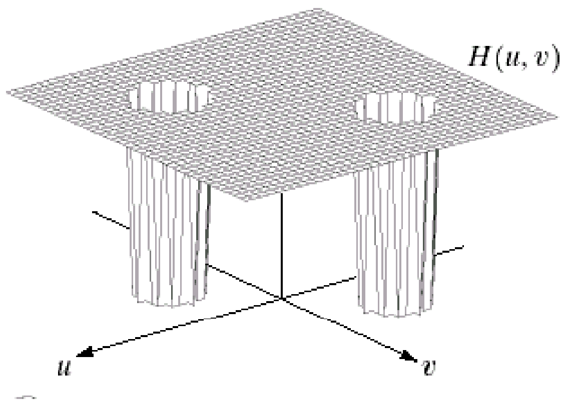
Restoration – in Frequency domain

- Band-reject Filter



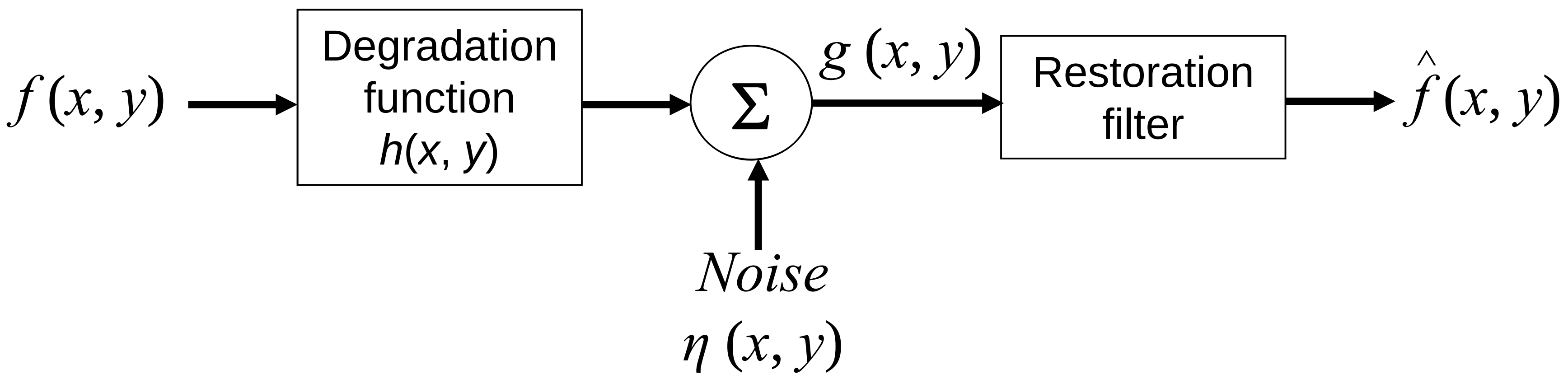
Ideal Butterworth Gaussian

- Notch Filter



Please check slides of last topic for more examples…

Estimating degradation function



Model of linear, position-invariant degradation

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v) \times F(u, v) + N(u, v)$$

Estimation by observation

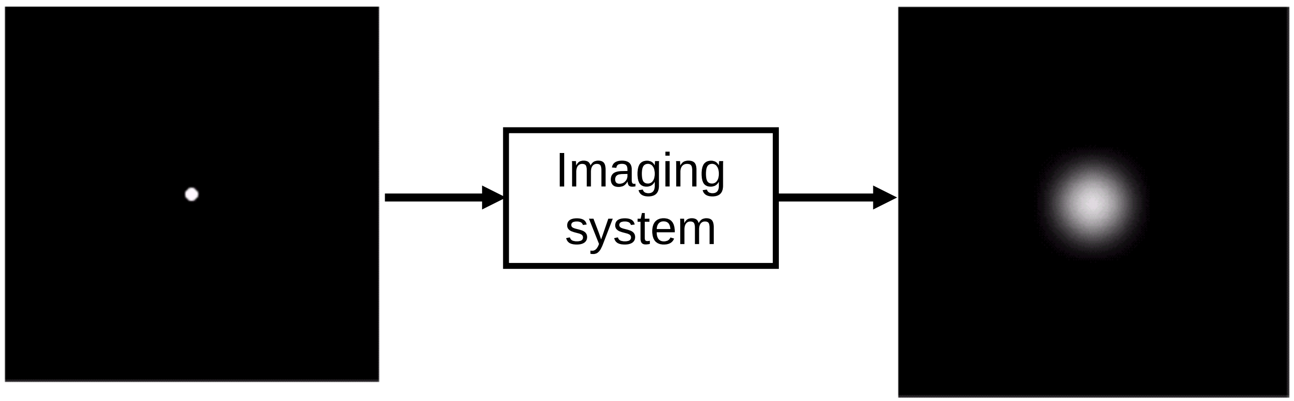
- Observation
  - 取影像中具有強烈訊號的子影像 $g_s(x,y)$
  - 建立與 $g_s(x,y)$ 相同大小的ideal image $\hat{f}_s(x,y)$
  - 假設 noise 可忽略:

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

品質取決於 $\hat{f}_s$，應用範圍有限！

Estimation by experimentation

- Experimentation
  - 對點光源 (越亮、越小越好) 取樣 (strength of the impulse: *A*)
  - $H(u,v) = \dfrac{G(u,v)}{A}$



Impulse of light $g\,(x, y)$

Estimation by modeling

- Derive a mathematical model according to the basic principles.
  - To know how image was degraded first.
  - Describe the model mathematically.

Estimation by modeling

- Modeling
  Ex: 大氣紊流

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

where *k* depends on the nature of the turbulence.

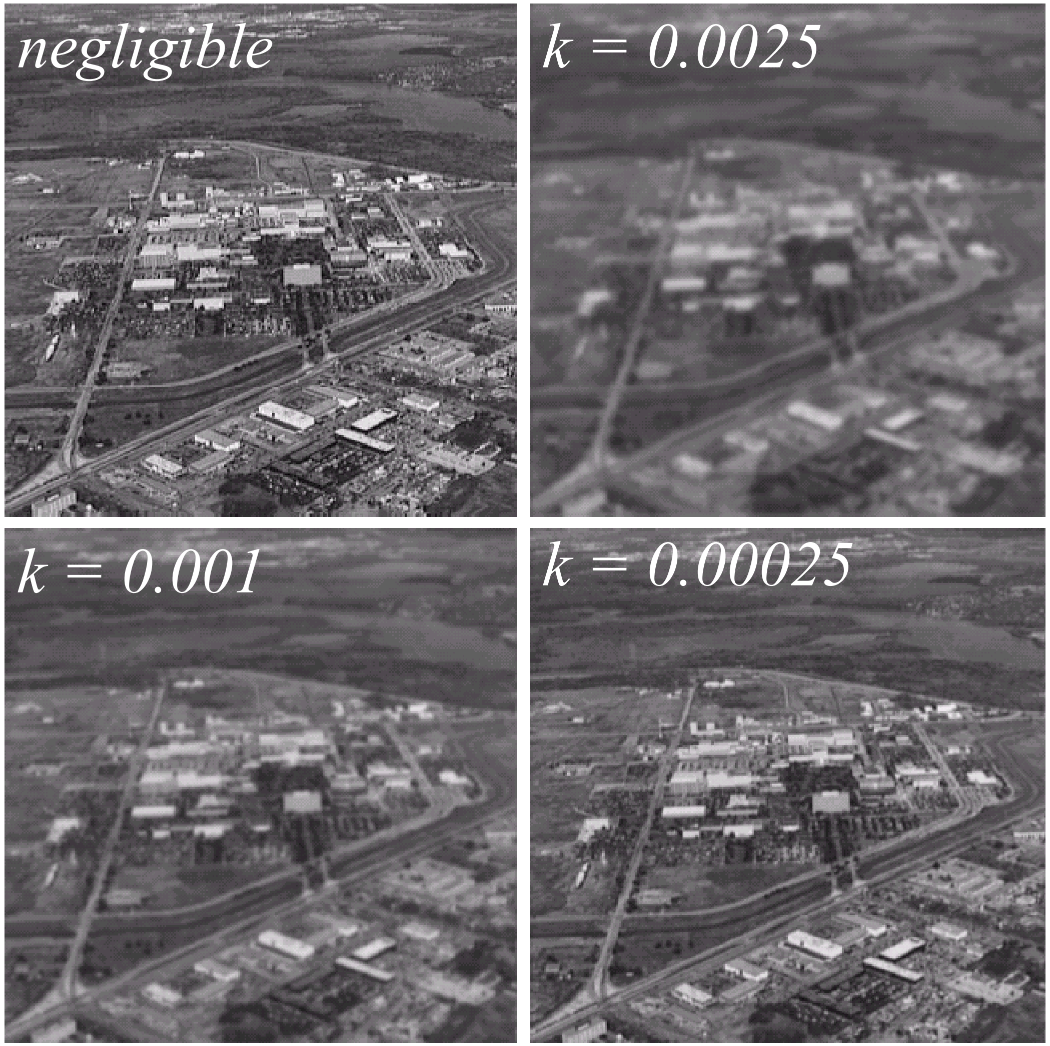


Image restoration approaches

- Inverse filtering

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

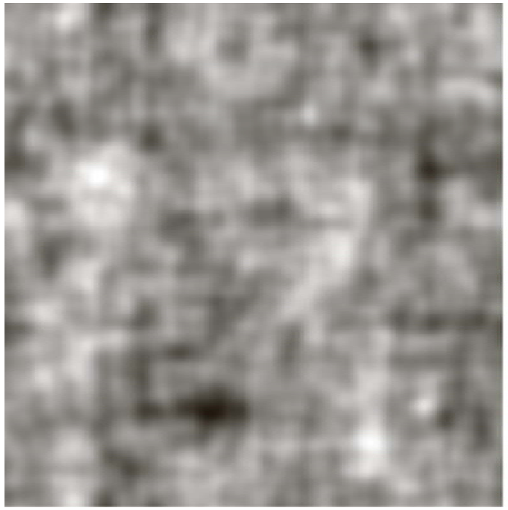
$$\hat{F}(u,v) = F(u,v) + \boxed{\frac{N(u,v)}{H(u,v)}}$$

- Even *H* was precisely estimated,…
- When *H* has zero or very small values,…

A 480 × 480 image degraded by a function with *k = 0.0025*

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

Direct inverse filtering



Cut-off freq.= 40

Cut-off freq.= 70

Cut-off freq.= 85

Wiener filtering

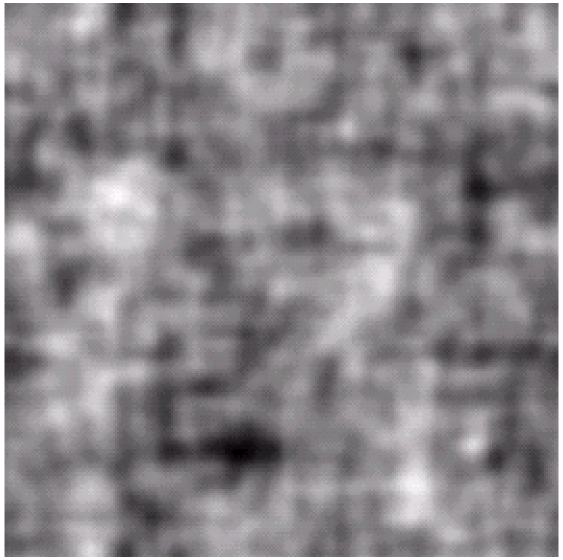
- Least mean square error filtering
  - 最小平均平方誤差濾波
- To minimize the error function
  - $e^2 = E\{(f - \hat{f})^2\}$

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] \cdot G(u,v)$$

$S_\eta(u,v)/S_f(u,v)$ → **~ 1/*SNR***

$S_\eta(u, v) = | N(u, v) |^2$: power spectrum of the noise
$S_f(u, v) = | F(u, v) |^2$: power spectrum of the undegraded image

Wiener filtering



Direct inverse filtering

Inverse filtering with a cut-off freq. of 70

Wiener filtering

Wiener filter is also an adaptive filter.

Left column: Images corrupted by motion blur and additive noise.

加入的雜訊強度減小

Direct inverse filtering *Wiener filtering*

Review

- Model of image restoration
  - Noise model
- Noise-only: restoration filters
- Degradation functions
  - Estimates of *H*
  - Approaches to restore images

生醫影像研究方法：影像復原